Millikan Experiment

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Contents

1 Abstract

The purpose of this lab is to use various physical variables related via equations of motion in order to experimentally determine the fundamental charge of an electron. In this lab, we essentially observed electrically charged (due to friction) droplets of oil fall within the dielectric (in this case, air) of a capacitor, changing the applied voltage on the conducting plates so as to manipulate the force exerted on the droplets and influence their motion. According to our results, the charge of an electron was calculated to be $1.3058 \cdot 10^{-19}$C $\pm 6.8 \cdot 10^{-22}$ C, which is quite different than the expected value of $1.6022 \cdot 10^{-19}$ C. However, this discrepancy, which is still within an order of magnitude of the expected value, can be attributed to limitations in the lab apparatus as well as errors in experimental measurements; the procedure and insights gained from the lab are still valid.

2 Introduction

The Millikan Oil Drop experiment plays a critical role in scientists' understanding of chemistry and physics in that it seeks to approximate a fundamental property of the microscopic world—the charge of an electron [1]. The expected value of this elementary charge, e, is $1.6022 \cdot 10^{-19}$ C [2]. The purpose of this experiment is to determine, via gravitational, electric, and drag forces, the fundamental charge of an electron. In this experiment, oil droplets are pushed out of an atomizer, causing them to become electrically charged due to friction [3]. The charged oil droplets then fall between the conducting plates of a capacitor (in this case, the capacitor's dielectric is air), where a combination of gravitational, drag, and/or electric forces act on the droplets. The magnitude of the electric force is directly proportional to the magnitude of the generated electric field, which is in turn, dependent on the magnitude of the applied voltage via the following equation:

$$E = \frac{V}{d} \tag{1}$$

where E is the magnitude of the electric field, V is the applied voltage across the conducting plates, and d is the distance between the plates. Another concept worthwhile introducing before stating the main equations of the lab is the equation detailing Stokes' resistance force for moving spherical objects. This viscous force that acts on the oil droplets always opposes the direction of motion and it is the force responsible for maintaining droplets' terminal velocity. The drag force in this instance, under laminar flow conditions, is approximated by the following equation:

$$F_{drag} \approx 6\pi r \eta v \tag{2}$$

where F_{drag}, r, and v are the drag force acting on, the radius of, and the speed of the oil droplets respectively, and η is the the dynamic viscosity of air at 1 atm [4]. By varying the applied voltage, the oil droplets obey one of three equations of motion depending on the following conditions: Equation 3 is obeyed if the net force on the droplet is 0, Equation 4 is obeyed if the droplet is moving downward with the terminal velocity, Equation 5 is obeyed if the motion is upward and the droplet is accelerating. The corresponding motions of equation are as follows:

$$g(m_{oil} - m_{air}) - \frac{QV_{stop}}{d} = 0 \tag{3}$$

where g is the acceleration due to gravity, m_{oil} and m_{air} are the masses of oil and air respectively, Q is the electric charge of the electron, V_{stop} is the applied voltage necessary to cause the droplet to experience a net force of 0, and d is the distance between the conducting plates.

$$g(m_{oil} - m_{air}) - 6\pi r \eta v_t = 0 \tag{4}$$

where m_{oil} and m_{air} are the masses of oil and air respectively, r is the radius of the oil droplet, η is the viscosity of air at 1 atm, and v_t is the terminal velocity of the droplets.

$$ma = -g(m_{oil} - m_{air}) + QE - 6\pi r \eta v \tag{5}$$

where m and m_{oil} are the masses of an oil droplet, g is the acceleration due to gravity, m_{air} is the mass of displaced air, Q is the electric charge of the electron, E is the electric field produced by the capacitor, r is the radius of the oil droplet, η is the viscosity of air at 1 atm, and v is the velocity of the oil droplet. It is worth noting that finding the masses of individual oil droplets is impractical considering the scope of this course as well the level of available equipment. Coupled with the assumption that the oil droplets are spherical in shape, the masses of these droplets can be alternatively expressed by manipulating the formula for density ($\rho = \frac{m}{V}$) to result in the following equation

$$m = \frac{4}{3}\pi r^3 \rho \tag{6}$$

where m is the mass of an object, r is its radius, and ρ is its density. In order to calculate Q, two methods are used in this lab. The actual conditions under which each equation is used are explained in Section 4 of this report (section outlining the lab procedure), but the equations corresponding to methods 1 and 2 are as follows respectively

$$Q = \frac{18\pi d \eta^{\frac{3}{2}}}{\sqrt{2g(\rho_{oil} - \rho_{air})}} \frac{v_t^{\frac{3}{2}}}{V_{stop}} = const_1 \frac{v_t^{\frac{3}{2}}}{V_{stop}} \tag{7}$$

and

$$Q = \frac{18\pi \eta^{\frac{3}{2}} d}{\sqrt{2g(\rho_{oil} - \rho_{air})}} (v_t + v_2) \frac{v_t^{\frac{1}{2}}}{V_{up}} \tag{8}$$

$$Q = const_2 (v_t + v_2) \frac{v_t^{\frac{1}{2}}}{V_{up}} \tag{9}$$

where v_2 is the velocity of the oil droplet and all other variables are as defined above. The derivations for these equations can be found in Section 8.1 of the Appendix. An equation that was useful in deriving Equations 7 and 9, relating the radius of a droplet to a combination of other variables, is as follows

$$r = 3\sqrt{\frac{\eta v_t}{2g(\rho_{oil} - \rho_{air})}} \tag{10}$$

Equation 10 was derived from manipulating Equation 4 and subbing in Equation 6.

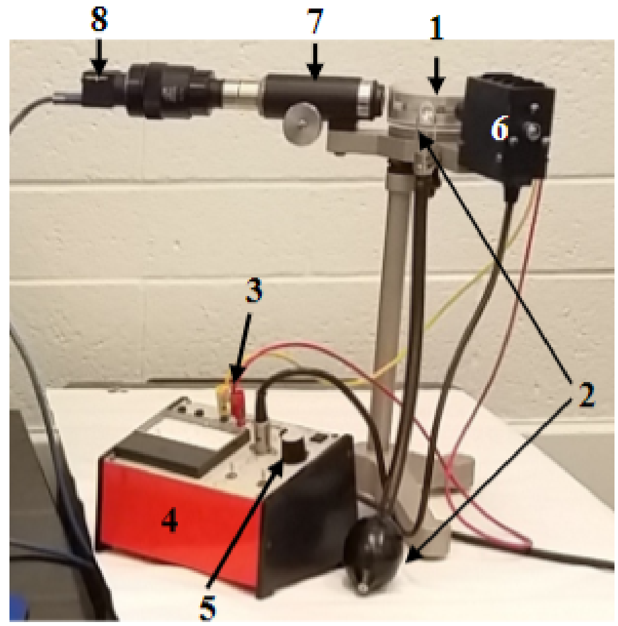

Figure 1: Leybold-Heraeus apparatus. 1 - Plate capacitor, 2 - Oil atomizer, 3 - Socket pair for charging the plate capacitor, 4 - Voltage source, 5 - Voltage adjustment knob, 6 - Light source, 7 - Microscope, 8 - Camera

3 Materials and Experimental Setup

The main pieces of equipment that were used in this experiment were the Leybold-Heraeus apparatus which allowed us to move oil droplets through the capacitor's dielectric and the LabVIEW software which tracked the individual oil droplets' locations as they travelled [3]. The camera on the apparatus was set up so that it had a calibration factor of (520 ± 1) px/mm and a frame rate of 10 Hz. The spray nozzle of the apparatus was positioned before the small holes in the acrylic glass cover of the Millikan chamber so as to ensure that a sufficient amount of oil droplets could be observed. Using the LabVIEW software, we were able to track individual oil droplets and increase the visibility of droplets so as to increase the accuracy of our data.

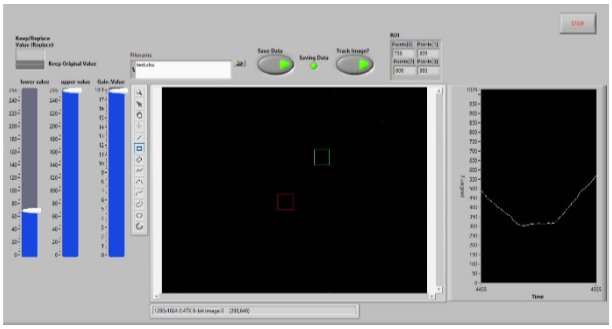

Figure 2: LabVIEW software UI. Red and green boxes display the software's ability to track oil droplets; graph on the right corresponds to a droplet's motion

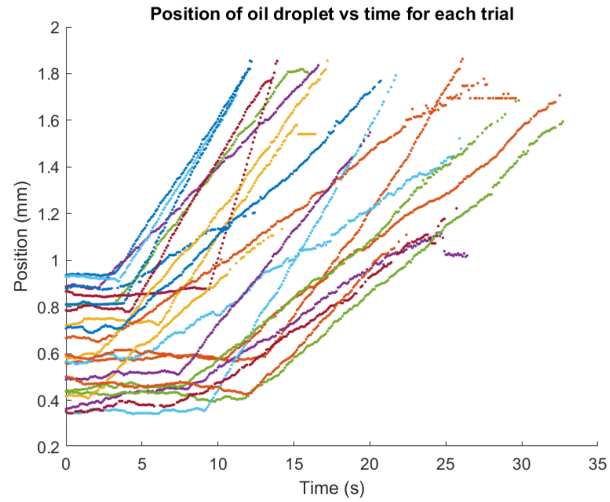

Figure 3: Recorded position vs time data for different trials.

4 Procedure

The voltage was set close to its maximum value. The upper gain value was set to its maximum value and the lower gain value was adjusted in order to optimize display illumination. Method 1 described in the lab manual was used for all trials. Prior to each trial, an Excel sheet was created with the trial number and stop voltage used for that trial. For each trial, the rubber bulb was squeezed to let oil droplets into the chamber and a droplet was selected to be tracked. The voltage was adjusted until the droplet stopped falling and was recorded as the stop voltage V_{stop}. Using the mouse, a green frame was drawn around the droplet using the LabVIEW software to track it. The voltage was quickly turned off and the software tracked the droplet until it moved out of frame. Data from each trial was then saved into its corresponding Excel sheet.

5 Results

After recording the results in Excel sheets, we used the scaling factor of 520 px/mm and frame rate of 10 Hz stated in the lab manual to convert the values to the correct units. Plotting the position vs time graphs of each trial on a graph, we obtain a plot shown in Figure 3. Note that several trials were omitted due to the low quality of the recorded data. Only data from 23 trials (cut down from 33 originally) remain in the plot in Figure 3.

Since Method 1 was used in the experiment, we use Equation 7 to calculate each trial's value of the droplet's charge, Q. Since Method 1 is a special case of Method 2, we assume that the upwards velocity v_2 in Equation 9 is zero and so the only values that needed to be extracted from the recorded data was the terminal velocity v_t.

In order to find the terminal velocity v_t, the sloping portion of each trial's data needed to be linearly fitted. Then v_t would be given by the slope of the linear fit. To fit the sloping portions, it was necessary to get rid of the data points from each trial where the oil droplet is stationary, i.e. the portion of the graph where the slope is flat. We cut the data manually, simply by observing when the data began to slope upwards in a linear fashion. To lessen the effect on accuracy, we cut the data fairly liberally to ensure that we weren't including any portions where the oil droplet was still accelerating. After extracting data from the sloped portion of each trial, we used MATLAB (example code found in Section 8.2) to linearly fit the data provide the values of the slope. An example of a trial's linear fit is shown in Figure 4.

After applying a linear fit to all trials and recording the calculated slope v_t and their corresponding stopping voltage V_{stop}, Equation 7 was

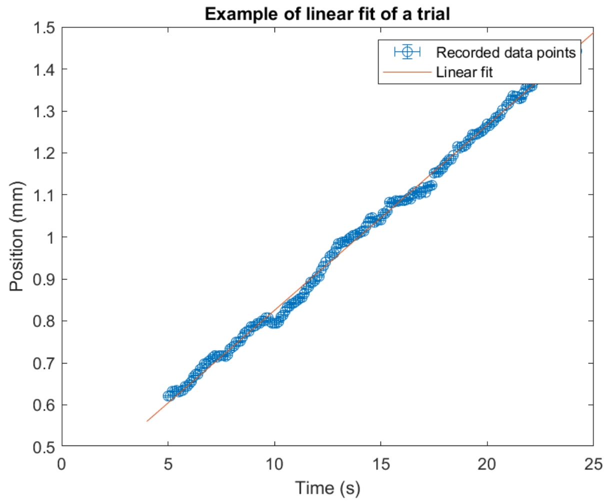

Figure 4: An example of the linear fit for the trial with $V_{stop} = 44$ V. The error in position is given by converting the ± 1 px error into millimetres, giving an error of $\pm$ 0.00192 mm. Error bars are too small to be seen on the plot.

used to calculate values of Q (one for each trial). We then used a Python script (the code can be found in Section 8.3) to calculate the greatest common divisor of all the calculated Qs, which gives an approximated value of the elementary charge, e. Our resulting calculated value of the elementary charge is

$$e = 1.3058 \cdot 10^{-19}\mathrm{C} \pm 6.8 \cdot 10^{-22}\mathrm{C}. \tag{11}$$

The calculation of the uncertainty is discussed in Section 6.1.

5.1 Goodness of Fit Analysis

5.1.1 R-squred method

We first test goodness of fit using the R-squared method. Using MATLAB's *fitlm* linear regression model, we were able to quickly obtain the R-squared value for each fit (results found in Section 8.4). All of the R-squared values were extremely close to the ideal value of 1; values were generally in the range of 0.9 to 1. This indicates that all of the linear fits were extremely good for the recorded data.

5.1.2 Standard Error

We also used standard error to analyze the goodness of fit for each trial. Using MATLAB's built in *std* function, we calculated the standard error for each trial's data (results found in Section 8.4). All of the values for standard error were extremely close to 0; they were within one uncertainty ($\pm$0.000192 mm) of zero. Therefore, based on standard error, the linear fits were determined to be good fits for the data.

6 Discussion

6.1 Uncertainties and Accuracy of Results

To calculate the uncertainty in the calculated value of e, the following error propagation formula was used:

$$\delta Q = |Q|\sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \ldots} \tag{12}$$

where Q is the value of the quantity you are calculating error for, δx represents the uncertainty in some variable x, and $a, b, \ldots$ are variables used in the calculation of the final value.

Our calculated value of the elementary charge does not agree with the expected value of $1.6022 \cdot 10^{-19}$ C. There were multiple sources of error throughout the experiment that could have contributed to this discrepancy. One is the excess light in the room when collecting the data – it made tracking the oil droplets significantly more difficult and could have produced erroneous data. Another is that, due to the fact we chose to use Method 1 to calculate Q, it is possible that we unreasonably assumed that v_2 from Equation 9 was zero. In reality, the slope of the flatter portions of the position vs time graphs weren't perfectly flat and the oil droplets could have been moving upwards with an unnoticeable velocity to the human eye, but could be significant in later calculations.

6.2 Numerical Values of $const_1$ and $const_2$ for Equations 7 and 9

Using the following provided values, the $const_1$ and $const_2$ terms in Equations 7 and 9 can be calculated: $\rho_{oil} = 875.3\frac{kg}{m^3}$, $\rho_{air} = 1.204\frac{kg}{m^3}$, $g = 9.80\frac{m}{s^2}$, $\eta = 1.827 * 10^{-5} Pa \cdot s$, $d = 6.0mm$. Plugging these values in Equations 7 and 9, we find that

$$const_1 = const_2 = 2.024 * 10^{-10} N \cdot s^{\frac{3}{2}} \cdot m^{\frac{-1}{2}}$$

6.3 Estimate of the radius of typical droplets

To estimate the radius of typical oil droplets, we can use Equation 10, subbing in the constant values η, g, ρ_{oil}, ρ_{air}, as well as our mean value for terminal velocity v_t. Consequently, the radius of typical droplets is estimated to be

$$r = 3\sqrt{\frac{1.827*10^{-5}(0.000079118)}{2(9.8)(875.3-1.204)}} \approx 8.7141 \cdot 10^{-7} m$$

6.4 Significance of buoyant force

The buoyant force is given by the following equation:

$$F_B = -m_{air} g = -\frac{4}{3} \tag{13}$$

Since the radius of the oil droplets is so small and the r term is cubed when calculating F_B, the magnitude of the buoyant force is extremely low. However, because the calculated values in this lab are all fairly small, the buoyant force should be accounted for in order to ensure that collected data is as accurate as practically possible. Thus, due to the microscopic nature of this lab, the buoyant force, though small, is still slightly significant.

6.5 Experimenting with different radii

This experiment works better when working with larger radii. Using Equation 10 we were able to estimate the radii of the oil droplets from each trial. When plotting the position vs time graphs, it was noticed that for larger radii, there were significantly less fluctuations in the data collected and the data overall seemed to follow a much smoother line, producing smaller error values. We theorize that this could be a result of the oil droplet's larger mass. With a larger mass, other forces such as buoyancy and gravitational forces have a larger impact on the droplet, therefore reducing the relative impact of the electric force on the oil droplet's motion.

7 Conclusions

Our calculated value for an electron's charge was $Q = 1.3058 \cdot 10^{-19}$C $\pm 6.8 \cdot 10^{-22}$ C. The percent error, relative to the accepted value of $1.6022 \cdot 10^{-19}$ C, is about 15.7%. Although it seems as if our calculated value is fairly inaccurate when compared to the expected value, it is important to remember how small the fundamental charge of an electron is; the fact that our calculations and the expected value are within similar magnitudes is of note in itself. Furthermore, there are multiple sources of error that could have contributed to the discrepancy between our calculated and the expected value, including excess light in the room when collecting data and the potentially inaccurate assumption that the upwards velocity v_2 was zero. However, to enhance the accuracy of our results for future instances of this lab, we could capture more data points, since gathering a larger range of data could reduce statistical uncertainty. Furthermore, as explained in Section 6.5, this experiment is better done with oil droplets of larger radii. Thus, lab results may improve if an atomizer with slightly larger holes was used, thus increasing the size of oil droplets by a tiny, but significant amount.

8 Appendix

8.1 Derivations of Equations 7 and 9

Of note for both derivations: Equations 6 and 10 were used to express the mass of oil and air in terms of their densities and radii since calculating the masses of individual oil and air particles is fairly difficult with the level of lab equipment for this lab.

Exercise 1 (Derivation of Equation 7

Isolate Equation 3 to obtain the following expression for Q:

$$Q = \frac{g(m_{oil} - m_{air})d}{V_{stop}} \tag{14}$$

Isolate Equation 4 to obtain the following expression for r:

$$r = \frac{g(m_{oil} - m_{air})}{6\pi\eta v_t} \tag{15}$$

Substitute Equation 6 into m_{oil} and m_{air} in Equation 15 to obtain the new expression for r (note that the following equation is just Equation 10):

$$r = 3\sqrt{\frac{\eta v_t}{2g(\rho_{oil}-\rho_{air})}}$$

Equation 14 can be rewritten in the following form after substituting Equation 6 into the m_{oil} and m_{air} terms

$$Q = \frac{4g\pi dr^3(\rho_{oil} - \rho_{air})}{3V_{stop}} \tag{16}$$

After subbing Equation 10 into Equation 16, the desired equation is derived (this is just Equation 7)

$$Q = \frac{18\pi d\eta^{\frac{3}{2}}}{\sqrt{2g(\rho_{oil}-\rho_{air})}}\frac{v_t^{\frac{3}{2}}}{V_{stop}}$$

The $\frac{18\pi d\eta^{\frac{3}{2}}}{\sqrt{2g(\rho_{oil}-\rho_{air})}}$ term in Equation 7 is a constant term with units $N \cdot s^{\frac{3}{2}} \cdot m^{\frac{-1}{2}}$.

Exercise 2 (Derivation of Equation 5)

Isolate Equation 5 to obtain the following expression for Q:

$$Q = \frac{(\frac{4}{3}g\pi r^3(\rho_{oil} - \rho_{air}) + 6\pi r\eta v_2)d}{V_{up}} \tag{17}$$

After substituting Equation 6 into Equation 17, the following simplified expression can be written:

$$(\frac{18\pi\eta v_t\sqrt{\eta v_t}}{\sqrt{2g(\rho_{oil} - \rho_{air})}} + \frac{18\pi\eta v_2\sqrt{\eta v_t}}{\sqrt{2g(\rho_{oil} - \rho_{air})}})\frac{d}{V_{up}} \tag{18}$$

Equation 18 can be rearranged in the following manner to derive the desired equation (this is just Equation 9)

$$\frac{18\pi\eta^{\frac{3}{2}}d}{\sqrt{2g(\rho_{oil}-\rho_{air})}}(v_t + v_2)\frac{v_t^{\frac{1}{2}}}{V_{up}}$$

The $\frac{18\pi\eta^{\frac{3}{2}}d}{\sqrt{2g(\rho_{oil}-\rho_{air})}}$ term in Equation 9 is a constant term with units $N \cdot s^{\frac{3}{2}} \cdot m^{\frac{-1}{2}}$.

8.2 MATLAB code

The MATLAB code written for curve fitting can be found here: https://drive.google.com/file/d/1cOKOOR8WdM1CrL4F9txuAp1ntuowPlMq/view?usp=sharing.

8.3 Python code

The Python script for finding the greatest common divisor can be found here: https://drive.google.com/file/d/1OCvMsZ6msjA-Matb5f5PmR-HgmCZE7pj/view?usp=sharing. The code is largely based on template code found here: https://www.geeksforgeeks.org/gcd-in-python/.

8.4 Calculated values from trials

Results from all trials can be found in this Google Sheet: https://

docs.google.com/spreadsheets/d/1sMJJarb4c3VgZ53b7jxh8RtN70SFS16z6iwI3uXQok8/edit?usp=sharing

9 References

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