Interference and Diffraction Lab

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1 Introduction

Experiments conducted in the 19$^{\text{th}}$ century showed that light passing through narrow slits will spread out behind the slit and form patterns when projected on to a screen. By having a sensor analyze the patterns of lasers and plotting light intensity against distance, interference and diffraction can be compared [1]. Analyzing diffraction and interference patterns of light is of importance in the realm of physics and more specifically in the conversation surrounding the wave-particle duality of light, since they seek to prove the wave nature of light [2]. When light passes through a slit, it diffracts and the angle to the minima is given by the following equation:

$$a \sin\theta = m'\lambda \qquad (m' = 1, 2, 3, ...) \tag{1}$$

where a is the slit width, θ is the angle from the center of the pattern to the "m'^{th}" minimum, λ is the wavelength of the light, and m' is the order of diffraction.
When interference of light results from its passage through two slits, the angle from the central maximum to side maximum is given by the following equation:

$$d \sin\theta = m\lambda \qquad (m = 0, 1, 2, 3, ...) \tag{2}$$

where d is the slit separation, θ is the angle the center of the pattern to the "m^{th}" maximum, λ is the wavelength of the light, and m is the order of interference.
By using concepts from the superposition of waves, the intensity of light $I(\phi)$ can be expressed as follows:

$$I(\phi) = I(0)(\frac{\sin(\phi)}{\phi})^2 \tag{3}$$

where $\phi = \frac{\pi a}{\lambda}\sin(\theta)$ and the variables are defined as in Equation 1.
Although only one portion of this lab includes calculations related to the Heisenberg Uncertainty Principle, there is value in briefly explaining the concept behind the principle before listing the relevant equations. Essentially, the Heisenberg Uncertainty Principle posits that for a (moving) particle, the more precise measurements become with respect to position, the less precise measurements become with respect to momentum (and vice versa) [3]. This general relationship is expressed by the following inequality:

$$\Delta y \cdot \Delta p \geq \frac{h}{4\pi} \tag{4}$$

where $h = 6.6262 \cdot 10^{-34} J \cdot s$ is Planck's constant, Δy is the uncertainty in the particle's position, and Δp is the uncertainty in the particle's momentum.
We can express the particle's uncertainty of velocity and momentum with the following the equations respectively:

$$\Delta v_y = c \sin\theta_1 \tag{5}$$

$$\Delta p_y = \frac{h}{\lambda}\sin(\theta_1) = \frac{h}{a} \tag{6}$$

where c is the speed of light, a is the slit distance, θ_1 is the angle of the first diffraction minimum, λ is the wavelength of the particle, and h is Planck's constant.
The uncertainty relation is given by the following equation:

$$\Delta p_y \cdot \Delta y = h \geq \frac{h}{4\pi} \tag{7}$$

Experimentally, the angle θ_1 can be calculated using the equation:

$$\tan(\theta_1) = \frac{l}{b} \tag{8}$$

where l is the half width of the central maximum of the sensor and b is the distance from the center of the sensor to the center of the slit.
By substituting equation 8 into equation 7, the following equation results:

$$\frac{a}{\lambda}\sin(\tan^{-1}(\frac{l}{b})) = 1 \tag{9}$$

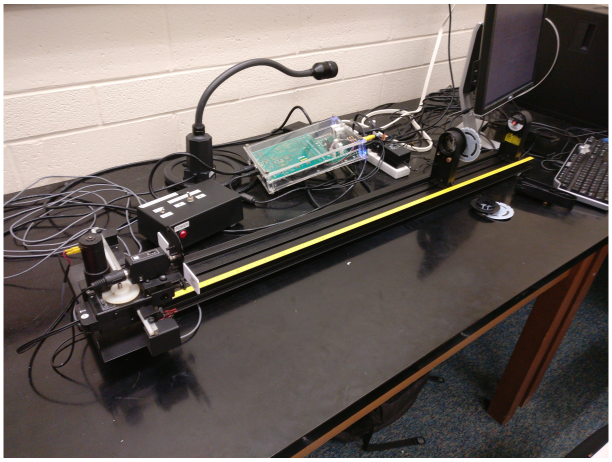

Figure 1: Experimental setup. The relevant equipment included in the photo are the motor, lens holder, laser beam source, and light sensor

Figure 2: Double slit disk.

2 Materials and Experimental Setup

- Single slit disk
- Multiple slit disk. The double slit setting was used.
- Scanner with Light sensor
- Red laser beam source ($\lambda = 650$ nm)
- Lens holder
- Motor
- Interference and Diffraction software

3 Procedure

3.1 Diffraction of Light Through a Single Slit

The lens holder was mounted with the single slit disk, using the 0.04mm slit. The laser, disk, sensor setup was arranged so that the laser pattern was displayed horizontally on the sensor. The laser pattern was then moved to one side of the sensor. The data acquisition button on the Interference and Diffraction software was clicked to turn it on. Then, the motor was turned on to move the laser pattern to the other side of the sensor. After reaching the other side of the sensor, the motor was turned off and the data acquisition button was clicked to turn it off.

3.2 Interference of Light Through Two Slits

The single slit disk was taken off of the lens holder and the multiple slit disk was mounted onto the lens holder. The disk was rotated so as to use a slit separation of $0.25mm$ and a slit width of $0.04mm$. The light aperture bracket on the sensor was set to slit #4. The methodology for data collection in this section is identical to the one outlined in section 3.1.

3.3 Quantum Mechanical Interpretation

Following the same setup and methodology as outlined in section 3.1, the half width of the central maximum for three different single slit widths was measured.

3.4 Diffraction Pattern Analysis

The lens holder was mounted with a single slit disk and the disk was adjusted to the 0.16mm slit setting. The same methodology as in section 3.1 was used. The pattern intensity corresponding to the central maximum was measured. The heights and corresponding angles of three secondary maxima were calculated.

Table 1: Calculated values of slit separation d from three different double slits. The slit type is specified as *slit width, slit separation* in mm.

Slit Type	d_{avg} (mm)	% difference
0.04, 0.25	1.43 ± 0.01	472%
0.04, 0.5	1.80 ± 0.04	261%
0.08, 0.25	1.49 ± 0.03	497%

4 Results

The values of intermediate results and calculations can be found in Appendix **??**. The calculations of all uncertainties is discussed in Section **??**.

4.1 Single Slit Exercise

By rearranging Equation **??**, we obtain a formula for the slit width a

$$a = \frac{m'\lambda}{\sin\theta} \tag{10}$$

$$\tag{11}$$

where θ can be calculated by measuring the distance between the the central maximum and local minima:

$$\theta = \arctan\frac{x}{l}. \tag{12}$$

Taking the distance l between the slit opening and the sensor to be 1.1 m and taking the average of two calculated values of a from two different minima, we get the the value of the slit width to be $a = 0.2070$ mm ± 0.0003 mm. This value is significantly different from the expected slit width value of 0.04 mm; the expected value is roughly five times smaller than the calculated value and does not lie within the calculated value's uncertainty.

4.2 Double Slit Exercise

4.2.1 Determining Slit Separation

Rearranging Equation **??**, we obtain a formula for the slit separation d

$$d = \frac{m\lambda}{\sin\theta} \tag{13}$$

where the value of θ is obtained in the same method as was explained in Section 4.1. The calculated average values of d and their percentage differences from the expected values are shown in Table 1. The expected values are significantly smaller than the calculated values and they do not lie within the calculated values uncertainties. Though it is worth noting that larger/smaller calculated slit separations correspond to larger/smaller expected slit separations.

4.2.2 Determining Slit Width

Using the exact same method as explained in Section 4.1, the slit widths of the three double slits discussed in Section 4.2.1 were calculated. The results are presented in Table 2. While the expected values do not agree with our calculated values and lie outside of their uncertainties, it is seen again that larger/smaller calculated slit widths correspond to larger/smaller expected slit widths.

Table 2: Calculated values of slit width a from three different double slits. The slit type is specified as *slit width, slit separation* in mm.

Slit Type	a (mm)
0.04, 0.25	0.1964 ± 0.0002
0.04, 0.5	0.2221 ± 0.0004
0.08, 0.25	0.4360 ± 0.0003

Table 3: Calculated values of the left side of Equation **??** for three different single slits. The value should theoretically be equal to 1 as discussed in Section **??**.

Slit Width a	Left side of Equation **??**
0.04 mm	0.1924 ± 0.0008
0.08 mm	0.1924 ± 0.0005
0.16 mm	0.1880 ± 0.0011

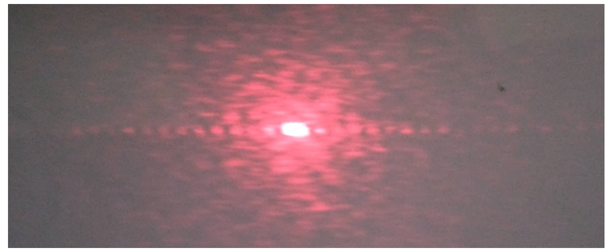

Figure 3: Light pattern produced when using a single slit width of 0.16 mm.

Table 4: Calculated values of θ_i corresponding to three different secondary maxima produced by a single slit width of 0.16 mm.

m	θ_i (degrees)
1	0.1855 ± 0.0008
2	0.2083 ± 0.0009
3	0.2171 ± 0.0013

4.3 Verifying Heisenberg's Uncertainty Principle

To verify Heisenberg's Uncertainty Principle, we show that Equation **??** holds. By measuring the half width of the central maximums (l) of three different single slit widths, we obtained three values for the left side of the equation presented in Table 3. These values should theoretically be equal to 1. The expected value of 1 was around five times greater than the calculated values and did not lie within their uncertainties. But by noticing that $1/4\pi \approx 0.0796$, we see that our results still verify Heisenberg's Uncertainty Principle since all of our calculated values are larger than $1/4\pi$.

4.4 Diffraction Pattern Analysis

By measuring and calculating the heights of three secondary maxima in the intensity pattern (shown in Figure 3) created by a single slit width of 0.16 mm, we calculated their corresponding angles θ_i, presented in Table 4.

To verify the intensity formula (Equation **??**), we used the exported data from the lab software and performed a non-linear fit using Python (the code used for curve-fitting can be found in Appendix **??**). The data and the fitted curve is shown in Figure 4. Beyond the rough overall shapes of the graphs, the fitted curve has little resemblance of the collected data. Although we are unclear on why the fit is so poor, we theorize that it may have something to do with how quickly the values in light intensity change.

4.4.1 Goodness of Fit Analysis

We first use the R-squared method to test the goodness of fit. Using the Scipy Python library, we were very easily able to calculate the R-squared value. A good fit should ideally have an R-squared value of 1. The value for our fit was $3.876 \cdot 10^{-5}$, which is extremely close to 0. This indicates that the fitted curve was not a good fit for the data.

We then use standard error to test the goodness of fit. Again using the Scipy library, we calculated the standard error of our fit. A good fit would ideally have a standard error close to 0. Our value for standard error was 0.000697 – a value close to zero, indicating a good fit. This

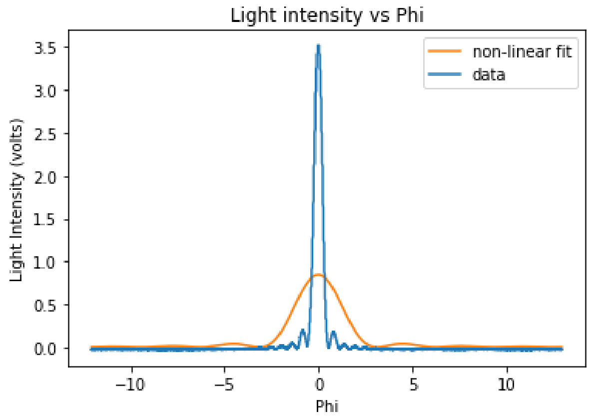

Figure 4: Data on light intensity plotted against ϕ and fitted to the intensity formula. The fitted value of $I(0)$ is $I(0) = 0.84438675$. Error bars are too small to be seen on the plot. Uncertainty in light intensity is $\pm 5 \cdot 10^{-6}$ V.

was surprising due to how bad the fit visually was. We theorize that the reason the value of standard error is so small is due to the ends of the graph where both the data and the fit are flat.

5 Discussion

5.1 Uncertainties

There were numerous sources of instrumental uncertainty throughout the lab that contributed to uncertainties in calculated values. The position measurements provided by the lab software were accurate to five decimal places, producing an uncertainty of $\pm 5 \cdot 10^{-6}$ m. The light intensity measurements were provided to the same precision, producing an uncertainty of $\pm 5 \cdot 10^{-6}$ V. When initially setting up for the experiment, a measuring tape was used to determine the distance between the slits and the light sensor. This measuring tape had an instrumental uncertainty of ± 0.0005 m.

For calculating the errors in calculated values, the following error propagation formula was used:

$$\delta Q = |Q| \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \ldots} \tag{14}$$

Here, Q is the value of the quantity you are calculating error for, δx is the uncertainty in some arbitrary variable, and a and b are variables used in the calculation of the final value.

5.2 What physical quantity is the same for the single slit and double slit?

Physical quantities that remain the same for single and double slits include the distance from the slits to the sensor and the slit width a.

5.3 Comparing the distance between the single-slit central maximum and first minimum and the double-slit central maximum and first diffraction minimum

The location of the first intensity minimum for a single slit and the location of the first diffraction minimum for a double slit are the same. Theoretically this is true according to Equation 1 (also mentioned in the lab manual), but experimentally we see that this holds true as well. Using a slit width of 0.04 mm for both single and double slits, the location of the first intensity minimum was measured to be consistently around 0.0034 mm $\pm$ 0.0002 mm.

5.4 What physical quantity determines where the amplitude of the interference peaks goes to zero?

The slit width a determines where the amplitude of the light intensity goes to zero. By examing the light intensity formula (Equation 3), we can see that as slit width a increases, the width of the central maximum decreases, and vice versa.

5.5 Theoretical number of interference maxima in the central envelope for a double slit with $d = 0.25$ mm and $a = 0.04$ mm

According to the lab manual, the first intensity minimum is theoretically located at

$$\theta = \arcsin\left(\frac{\lambda}{a}\right). \tag{15}$$

Plugging in $a = 0.04$ mm, we get that $\theta = 0.01625$ rad. Then, using Equation 2 and solving for m, we find that

$$\begin{aligned} m &= \frac{d\sin\theta}{\lambda} \\ &= \frac{0.00025\sin(0.01625)}{650\cdot 10^{-9}} \\ &= 6.25 \quad \text{interference maxima} \end{aligned}$$

Rounding down to a whole number, this means that there theoretically should be 6 maxima on *each side* of the central envelope. But this value includes the central maxima. Therefore theoretically there should be a total of 11 interference maxima in the central envelope.

5.6 Experimental number of interference maxima in the central envelope for a double slit with $d = 0.25$ mm and $a = 0.04$ mm

Experimentally, examining the graph produced by the lab software (shown in Figure 5, there are also 11 interference maxima located in the central envelope, agreeing with the result we obtained from Section 5.5.

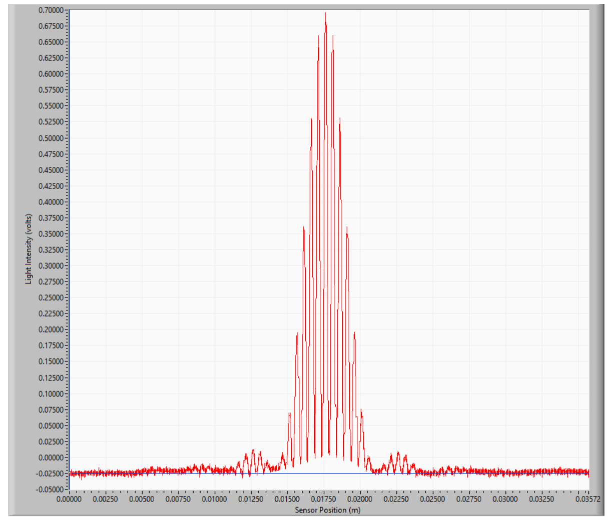

Figure 5: Graph produced when using a double slit width of 0.04 mm and slit separation of 0.25 mm.

6 Conclusions

Through this lab, it was experimentally verified that the location of the first diffraction minimum remains constant regardless of single slit or double slit since its distance from the central maximum for both cases remained within a narrow range of 0.0034 mm ± 0.0002 mm. We were unable to verify the light intensity formula (Equation 3 due to how poorly the non-linear fit was for the data. Furthermore, many of the results obtained from Equations 1 and 2 were several multiples off from the expected values. This leads us to believe that there might have been a systematic error in our set up. In order to improve the quality of data in the future, we should more carefully set up the experiment and more trials should be performed.

7 Appendix

7.1 Data and Code

All data, calculations, and code used in this lab are provided in the following Google Drive link: https://drive.google.com/drive/folders/1aTUZfrNO2RqHY3M3lkasRnt8EfrDKzKe?usp=sharing.

8 References

References

[1] R. M. Serbanescu, *Lab manual: Interference and Diffraction of Light*. 2008.

[2] S. Baron. "Curious kids: Is light a wave or a particle?" (2021), [Online]. Available: https://theconversation.com/curious-kids-is-light-a-wave-or-a-particle-162514.

[3] S. Exchange. "What is the uncertainty principle and why is it important?" (), [Online]. Available: https://scienceexchange.caltech.edu/topics/quantum-science-explained/uncertainty-principle.