AC CIRCUITS

CHAPTER 4 – CIRCUIT THEOREMS

NODAL ANALYSIS

1. Ground and identify nodes
2. Represent unknown variables in terms of node voltages
3. Write KCL at each node
4. Solve system of equations for node voltages

Nodal Analysis with Voltage Sources:

- Case 1: Voltage source between a node and ground
    - Set voltage at node equal to voltage of voltage source
- Case 2: Voltage source between two nodes
    - Two nodes become a SUPERNODE
    - Apply KCL to the supernode (like with other nodes), then get supplementary equation from inside the supernode

MESH ANALYSIS

1. Assign mesh currents to meshes
2. Represent unknown variables in terms of mesh currents
3. Apply KVL to each mesh
4. Solve system of equations for mesh currents

Mesh Analysis with Current Sources:

- Case 1: Current source exists in only one mesh
    - Set mesh current equal to current source
- Case 2: Current source exists between two meshes
    - Two meshes become a supermesh
    - Apply KVL to supermesh (like with other meshes), then get supplementary equation from inside the supermesh

SOURCE TRANSFORMATION (THEVENIN-NORTON TRANSFORMATION)

Replacing a voltage source in series with a resistor by a current source in parallel with a resistor (or other way around)

- $V_s = i_s R$
- Voltage and current must not conform to PSC
- Can also be applied to dependent sources

THEVENIN AND NORTON EQUIVALENT CIRCUITS

Thevenin

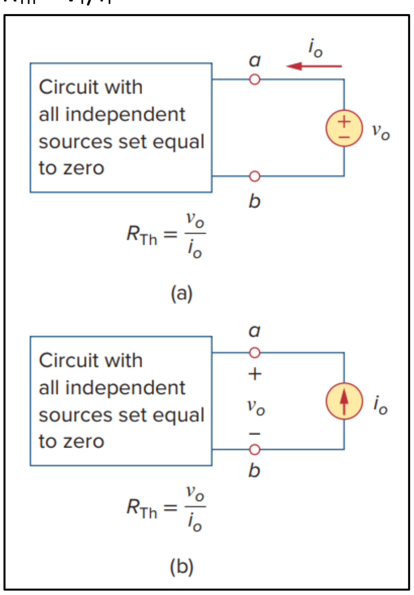
A linear, two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source in series with a resistor

Norton

A linear, two-terminal circuit can be replaced by an equivalent circuit consisting of a current source in parallel with a resistor

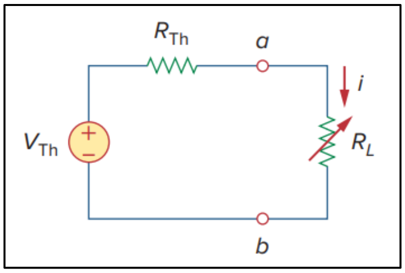
- V_{Th} = v_{oc} = open circuit voltage at terminals
  - Found using regular circuit analysis methods
- I_N = i_{sc} = short circuit current through terminals
- R_{Th} = R_N = equivalent resistance at terminals when independent sources are turned off

Methods for finding R_{Th}

1. Circuit has no dependent sources
   a. Deactivate all independent sources, find R_{eq}
2. Circuit has >= 1 independent source
   a. $R_{Th} = v_{oc}/i_{sc}$
   b. v_{oc} and i_{sc} must conform to PSC
3. Any circuit
   a. Deactivate all independent sources
   b. Connect a test current/voltage source (1 A or 1 V)
   c. Find voltage across current source or current across voltage source
      i. PSC must not be held
   d. $R_{Th} = v_T/i_T$
   e. 

*Note: if there are no independent sources, v_{oc} and i_{sc} = 0, so equivalent circuit is just a resistor

MAXIMUM POWER TRANSFER



Maximum power is transferred to the load when load resistance R_L equals Thevenin resistance seen from the load ($R_L = R_{Th}$)

- Maximum power transferred: $p_{max} = V_{Th}^2/4R_{Th}$

SUPERPOSITION PRINCIPLE

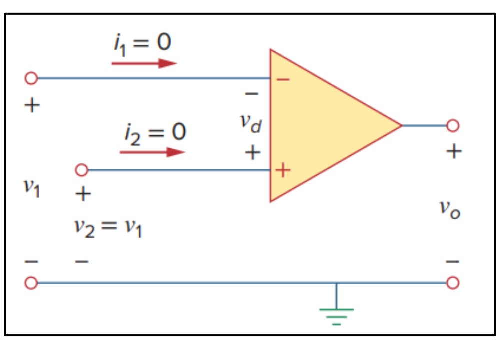
The voltage across/current through an element (in a linear circuit) equals the sum of voltages across/currents through tat element due to each independent source in the circuit acting alone

1. Turn off all independent sources except one. Find output voltage/current due to the active source.
    a. Voltage source = short circuit, current source = open circuit
2. Repeat for all other independent sources
3. Find total voltage/current by adding up individual contributions

CHAPTER 5 – OPERATIONAL AMPLIFIERS (IDEAL OP-AMPS)

2 Important Properties:

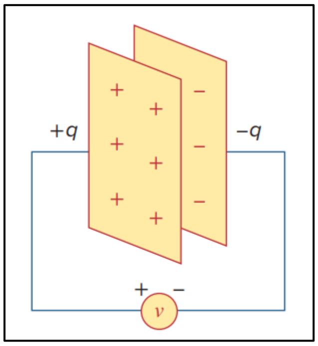
1. Currents at input terminals are zero, $i_1 = i_2 = 0$ A
2. Voltage across input terminals is zero, $v_1 = v_2$



CHAPTER 6 – CAPACITORS AND INDUCTORS

- They are passive <u>storage elements</u> – store energy to be retrieved at a later time (instead of dissipating like resistors)

CAPACITOR



- Capacitor: two plates separated by an insulator, stores energy in electric field between plates
- A connected voltage source deposits positive charge on one plate and negative on the other
    - Capacitor stores this electric charge
- Amount of charge stored on plates is proportional to applied voltage
- Constant of proportionality, C = <u>CAPACITANCE</u> (units: F [=] coulomb/volt)

$$q = Cv$$

CURRENT-VOLTAGE relation at a capacitor (assuming PSC holds): $\boxed{i = C\frac{dv}{dt}}$

VOLTAGE-CURRENT relation at a capacitor: $\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)}$

Instantaneous POWER delivered to capacitor: $\boxed{p = vi = Cv\frac{dv}{dt}}$

ENERGY stored in the electric field between plates of capacitor: $\boxed{w = \frac{1}{2}Cv^2}$ $\boxed{w = \frac{q^2}{2C}}$

<u>PROPERTIES of a capacitor</u>:

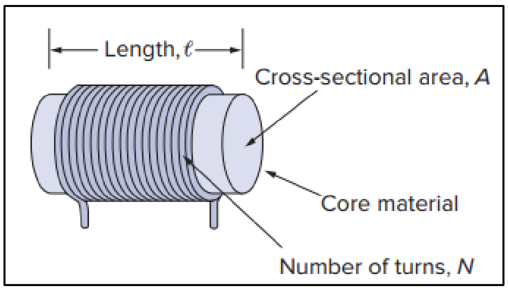
1. When voltage across capacitor is constant (steady-state DC condition), a capacitor is an open circuit
2. Voltage of a capacitor is continuous – cannot change abruptly
    a. Since voltage would then be non-differentiable
    b. But current can change abruptly

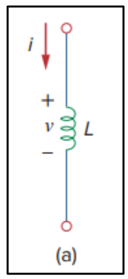
SERIES AND PARALLEL CAPACITORS

Parallel: $\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$

Series: $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$ $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

INDUCTOR





- Inductor: coil of conducting wire, stores energy in magnetic field
- Current pushed through an inductor creates a magnetic field that stores energy
    - If current being pushed through is constant, inductor behaves like wire
- If supplied current is abruptly changed, inductor's magnetic field generates a force to try to keep current flowing through it
- Inductors <u>resist instantaneous changes in current</u> flowing through it

VOLTAGE-CURRENT relation at an inductor (assuming PSC holds): $$v = L\frac{di}{dt}$$

- Where L is inductance (units: H)

CURRENT-VOLTAGE relation at an inductor: $$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$$

POWER delivered to inductor: $$p = vi = \left(L\frac{di}{dt}\right)i$$

ENERGY stored in an inductor: $$w = \frac{1}{2}Li^2$$

<u>PROPERTIES of an inductor:</u>

1. When current is constant, voltage across inductor is zero, so an <u>inductor is a short circuit under steady-state DC condition</u>
2. Current through an inductor cannot change instantaneously (but voltage across can)

SERIES AND PARALLEL INDUCTORS

- Exact same as resistors

SUMMARY TABLE

Important characteristics of the basic elements.[†]

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

[†] *Passive sign convention is assumed.*

CHAPTER 7 – FIRST-ORDER CIRCUITS

- Characterized by first order DEs.
- 2 ways to excite first-order circuits:
  - Source-free – energy initially stored in the capacitor/inductor
  - By independent sources
- "Circuit response" – how the circuit reacts to an excitation
  - Natural response – behaviour (in terms of voltages and currents) of a circuit with no external sources of excitation
- Apply KVL to get the first order DE:

SOURCE-FREE RC CIRCUITS

- DC source is suddenly disconnected – energy stored in capacitor is released to resistors
- Circuit response is its natural response (exponential decay of initial voltage):

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$

- $\tau = RC$, time constant (seconds): time required for response to decay to a factor of 1/e of initial value (larger tau = slower response)
  - R = R_{Th} at terminals of capacitor
- $i_C(t) = C\frac{dv}{dt} = -\frac{V_0}{R}e^{-\frac{t}{\tau}}$

Working with Source-Free RC Circuit:

1. Initial voltage V_0
2. Time constant $\tau = RC$

STEP RESPONSE OF AN RC CIRCUIT

- Step Response: behaviour of circuit due to a sudden application of a DC voltage/current source
  - Excitation is the step function u(t)

$$\text{Complete response} = \underbrace{\text{transient response}}_{\text{temporary part}} + \underbrace{\text{steady-state response}}_{\text{permanent part}}$$

- Transient response: part that decays to zero as t approaches infinity
- Stead-state response: part that is permanent

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \qquad v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau}$$

- $i_C(t) = C\frac{dv}{dt} = -\frac{V(\infty)-V(0)}{R}e^{-\frac{t}{\tau}}$

Finding Step Response:

1. Initial capacitor voltage
2. Final capacitor voltage
3. Time constant, $\tau = RC$

SOURCE-FREE RL CIRCUITS AND STEP RESPONSE OF AN RL CIRCUIT

- Same as RC circuits, except we solve for current instead of voltage
- Time constant $\tau = \frac{L}{R}$

Source-Free RL Circuits:

$$i(t) = I_0 e^{-t/\tau}$$

- $v_L(t) = L\frac{di}{dt} = -RI_0e^{-\frac{t}{\tau}}$

1. Initial current I_0
2. Time constant $\tau = \frac{L}{R}$

Step Response of RL Circuits

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \qquad i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-(t-t_0)/\tau}$$

1. Initial inductor current
2. Final inductor current
3. Time constant, $\tau = \frac{R}{L}$

AC CIRCUITS

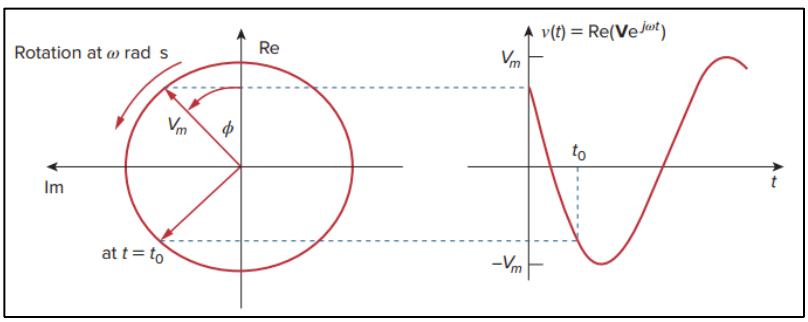
CHAPTER 9 – SINUSOIDS AND PHASORS

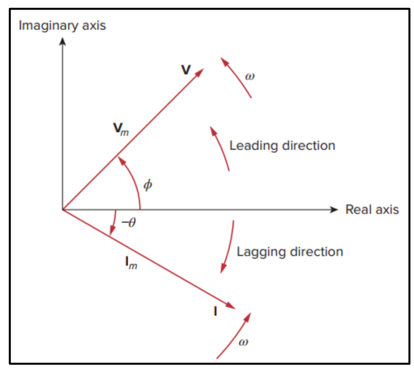
PHASORS

- Complex number that represents amplitude and phase of sinusoids

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \underline{/\phi}$$

(Time-domain representation) (Phasor-domain representation)





PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS

- Resistors: V and I are in phase
- Inductors: V leads I
- Capacitors: V lags I

TABLE 9.2

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

IMPEDANCE AND ADMITTANCE

- Impedance [Ohms]: opposition to flow of sinusoidal current
  - Essentially more general version of resistance

$$\mathbf{Z} = R \pm jX = |\mathbf{Z}|\underline{/\theta}$$

  - R = Resistance, X = Reactance
  - Resistors are purely resistive (only dissipate energy)
  - Inductors and Capacitors are purely reactive (just store and release later)

$$\mathbf{Y} = G + jB$$

- Admittance [Siemens]: reciprocal of impedance
  - G = Conductance, B = Susceptance

TABLE 9.3

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

CHAPTER 10 – SINUSOIDAL STEADY-STATE ANALYSIS

- Everything is literally the exact same

CHAPTER 11 – AC POWER ANALYSIS

EFFECTIVE/RMS (ROOT MEAN SQUARE) VALUE

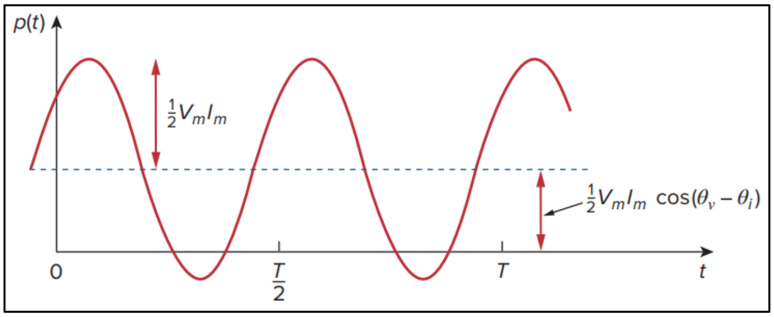
- DC value that delivers same amount of average power as AC current/voltage

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

AVERAGE POWER

- Average of instantaneous power over one period [Watts]
- General Equation:

$$P = \frac{1}{2}\text{Re}[\mathbf{VI}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$



  - Purely resistive (resistors, V and I are in phase, power is always absorbed):

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R \qquad P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

  - Purely reactive: P = 0
    - Capacitors and inductors, V and I have 90 deg phase shift, power is absorbed then generated

MAXIMUM AVERAGE POWER TRANSFER

- Find Thevenin equivalent circuit seen by variable load
- Maximum average power transfer occurs when reactance is neutralized as much as possible
- When R and X are both variable:

$$\mathbf{Z}_L = R_L + jX_L = R_{\text{Th}} - jX_{\text{Th}} = \mathbf{Z}_{\text{Th}}^*$$

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}$$

- When load is purely resistive (only R):

$$R_L = \sqrt{R_{\text{Th}}^2 + X_{\text{Th}}^2} = |\mathbf{Z}_{\text{Th}}|$$

- When R and X have constraints:
  - Find ideal X_L first (neutralize reactance of circuit as much as possible)
  - Use X_L to find ideal R_L:

$$R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2}$$

APPARENT POWER AND POWER FACTOR

- Apparent Power: what the power seems to be in DC (product of rms values of v and i) [VA]

$$S = V_{rms} I_{rms}$$

- Power Factor (pf): how much of the apparent power is actually dissipated (as average power)

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

  - Leading pf: current leads voltage (angle < 0) (implies capacitive load)
  - Lagging pf: current lags voltage (angle > 0) (implies inductive load)

COMPLEX POWER

- Contains all relevant power information for a load

$$\begin{aligned}
\text{Complex Power} = \mathbf{S} = P + jQ &= \mathbf{V}_{rms}(\mathbf{I}_{rms})^* \\
&= |\mathbf{V}_{rms}|\,|\mathbf{I}_{rms}| \underline{/\theta_v - \theta_i}
\end{aligned}$$

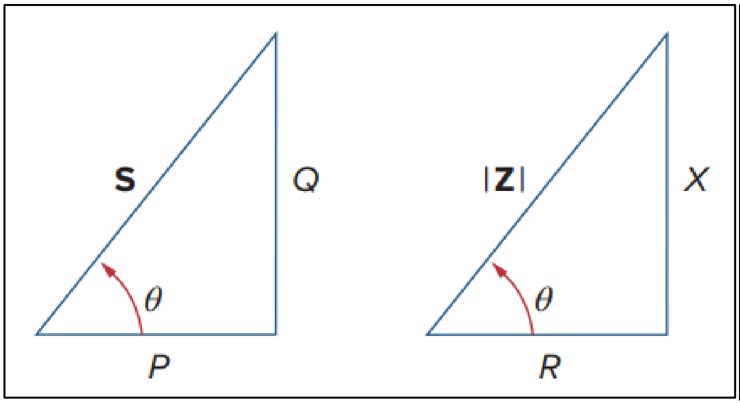
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{rms}|\,|\mathbf{I}_{rms}| = \sqrt{P^2 + Q^2}$$

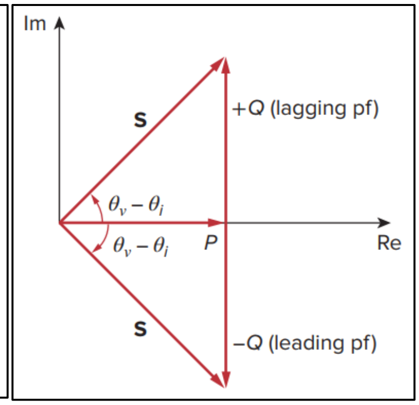
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

  - Real part of **S** is Real/Average power P
  - Imaginary part of **S** is Reactive power Q
  - Magnitude of **S** is Apparent power S
  - Cosine of phase angle is Power Factor pf
- Power Triangle, Impedance Triangle:





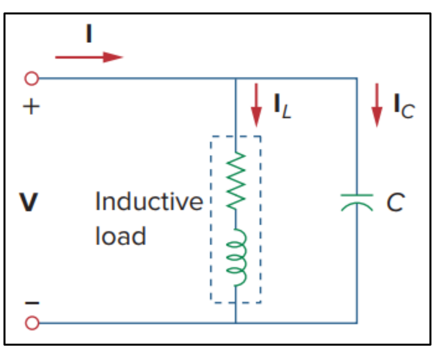
CONSERVATION OF AC POWER

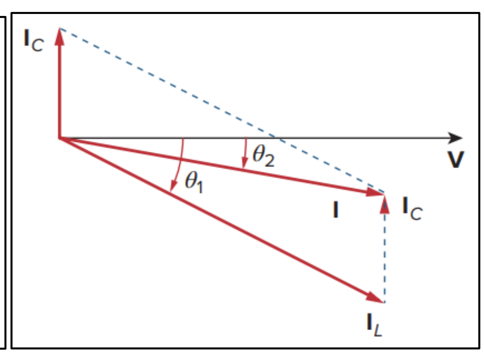
- Complex, real, and reactive powers of the sources equal sums of the complex, real and reactive powers of loads

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \cdots + \mathbf{S}_N$$

POWER FACTOR CORRECTION

- Process of increasing the power factor without altering voltage/current to load
  - The addition of a reactive element (usually capacitor) to make pf closer to 1
  - Choosing correct capacitor = I exactly in phase with V = minimize current = save money





- Equations that might be useful idk how they work lemao:

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

$$Q_L = \frac{V_{\text{rms}}^2}{X_L} = \frac{V_{\text{rms}}^2}{\omega L} \quad \Rightarrow \quad L = \frac{V_{\text{rms}}^2}{\omega Q_L}$$